A POINTWISE TRACKING OPTIMAL CONTROL PROBLEM FOR THE STATIONARY NAVIER–STOKES EQUATIONS

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Abstract. We study a pointwise tracking optimal control problem for the stationary Navier–Stokes equations; control constraints are also considered. The problem entails the minimization of a cost functional involving point evaluations of the state velocity field, thus leading to an adjoint problem with a linear combination of Dirac measures as a forcing term in the momentum equation, and whose solution has reduced regularity properties. We analyze the existence of optimal solutions and derive first and, necessary and sufficient, second order optimality conditions in the framework of regular solutions for the Navier–Stokes equations. We develop two discretization strategies: a semidiscrete strategy in which the control variable is not discretized, and a fully discrete scheme in which the control variable is discretized with piecewise constant functions. For each solution technique, we analyze convergence properties of discretizations and derive a priori error estimates.

Keywords: optimal control problem, Navier–Stokes equations, Dirac measures, first and second order optimality conditions, finite element approximations, convergence, error estimates.

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