A POLYTOPAL DISCONTINUOUS GALERKIN METHOD FOR THE APPROXIMATION OF BRAIN MULTIPHYSICS FLOW DYNAMICS

IVAN FUMAGALLI, MATTIA CORTI, NICOLA PAROLINI, AND PAOLA F. ANTONIETTI

Abstract. Several neurodegenerative diseases are associated with an impairment of the waste clearance mechanism carried out by the Cerebrospinal Fluid (CSF), which filtrates in the cerebral tissue and flows in brain ventricles to remove waste proteins. In this work, the CSF dynamics is modeled by a multiphysics model encompassing Multiple-Network Poroelasticity (MPE) equations for filtration and Stokes equations for the three-dimensional CSF flow in hollow regions [1]. The MPE equations also account for blood perfusion of the brain [2]: indeed, the production and washout function of CSF are significantly affected by blood pressure pulsatility [3]. To accurately represent the complex geometry of the brain and of the interface between the two physical domains, we introduce a discontinuous Galerkin method based on polytopal grids (PolyDG) for the spatial discretization of the model, for which we provide stability and convergence results [1]. We implemented this model in `lymph` [4], an open-source library developed at MOX for the solution of multiphysics problems with the PolyDG method. Simulations in patient-specific geometries reconstructed from magnetic resonance images show the suitability of the mathematical model and the effectiveness of the method.

Acknowledgments: IF, NP, and PFA acknowledge the funding by the Italian Research Center on High Performance Computing, Big Data and Quantum Computing (ICSC), under the NextGenerationEU project, and by MUR on the PRIN 2020 research grant n. 20204LN5N5. The authors are members of INdAM-GNCS and IF acknowledges the funding by the INdAM-GNCS Project CUP_E53C22001930001. We acknowledge CINECA for the availability of HPC resources under the ISCRA initiative. This work is part of the project Dipartimento di Eccellenza 2023-2027, Dipartimento di Matematica, Politecnico di Milano.

Keywords: Cerebrospinal fluid, Stokes' equation, Multiple-Network Poroelasticity Theory, Polygonal/polyhedral mesh, Multiphysics system

Mathematics Subject Classifications (2010): 65N12, 65N22, 65N30, 76Z05.

References

[1] I. Fumagalli, M. Corti, N. Parolini, and P.F. Antonietti. Polytopal discontinuous Galerkin discretization of brain multiphysics flow dynamics. Submitted. Preprint: https://arxiv.org/abs/2310.07651.

[2] M. Corti, P.F. Antonietti, L. Dede', and A. Quarteroni. Numerical modelling of the brain poromechanics by high-order discontinuous Galerkin methods. *Mathematical Models and Methods in Applied Sciences*, 33(8):1577-1609, 2023.

[3] M. Causemann, V. Vinje, and M.E. Rognes. Human intracranial pulsatility during the cardiac cycle: a computational modelling framework. *Fluids Barriers CNS*, 19:84, 2022.

[4] P.F. Antonietti, S. Bonetti, M. Botti, M. Corti, I. Fumagalli, and I. Mazzieri. lymph: an efficient high-order solver for differential problems with polytopal discontinuous Galerkin methods. Submitted.

MOX, Department of Mathematics, Politecnico di Milano
Email address: ivan.fumagalli@polimi.it

MOX, Department of Mathematics, Politecnico di Milano
Email address: mattia.corti@polimi.it

MOX, Department of Mathematics, Politecnico di Milano
Email address: nicola.parolini@polimi.it

MOX, Department of Mathematics, Politecnico di Milano
Email address: paola.antonietti@polimi.it