

# A MIXED-DIMENSIONAL MODEL FOR SIMULATING DIRECT CURRENT WITH A HIGH-RESISTIVITY LINER

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**ABSTRACT.** In this study, we introduce a mathematical model that combines different dimensions to analyze the electric potential and current density in direct current simulations, particularly when incorporating a thin liner within the modeled area. This liner is employed in landfill management to prevent leachate leakage from the waste body into the underground, and it is constructed from a highly impermeable, high-resistivity plastic material. Given that the electrodes and the liner possess significantly smaller diameters and thickness compared to their other dimensions, conducting numerical simulations for these elements in an equi-dimensional context could be prohibitively costly. Therefore, our approach involves approximating them as objects with reduced dimensions and subsequently deriving the corresponding equations. We validate the resulting mixed-dimensional model by comparing it to progressively complex laboratory experiments, demonstrating the reliability of our proposed mathematical framework. Our experiments also indicate that configurations involving current and voltage electrodes on opposite sides of the liner, enclosing the landfill, may be effective for detecting damage to the membrane.

**Keywords:** mixed-dimensional modeling, direct current simulations, sensitivity, laboratory comparison

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