BANACH SPACES-BASED FULLY MIXED FEMS FOR THE BOUSSINESQ PROBLEM WITH TEMPERATURE-DEPENDENT PARAMETERS

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Abstract. In this talk, we develop a family of Banach spaces-based approach yielding a fully-mixed finite element method to numerically solve the heat-driven flows with temperature-dependent viscosity modeled by the stationary Boussinesq equations. The momentum and energy conservation equations are formulated in terms of velocity and the tensors of strain rate, vorticity and stress, pseudoheat, temperature and its gradient; and the incompressibility constraint is used to eliminate the pressure, which is computed afterwards by a postprocessing formula depending on the stress and the velocity. The unique solvability of the continuous problem is addressed by invoking a global inf-sup property in an adequate abstract setting for perturbed saddle-point problems in Banach spaces and two assumptions further regularities on the solutions of the problem. Adopting an analogue approach for the associated Galerkin scheme, and under suitable hypotheses on arbitrary finite element subspaces employed, we show existence and then uniqueness of the discrete solution. In addition, the error analysis is conducted under appropriate assumptions on data and applying the first Strang lemma for the discrete solution as well as for the post-processed pressure. Finally, we report several numerical experiments.

Keywords: Boussinesq problem, temperature-dependent parameters, Banach spaces, fully mixed FEM, a priori error analysis.

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