A THIRD-ORDER UNCONDITIONALLY POSITIVE AND CONSERVATIVE MODIFIED PATANKAR RUNGE–KUTTA SCHEMES BASED ON OLIVER'S APPROACH, JANUARY 15 – 19, 2024

JAVIER GONZÁLEZ AND ANDRÉS ÁVILA

ABSTRACT. The mofified Patankar Runge-Kutta (MPRK) methods have proven to be efficient and robust numerical schemes that preserve positivity and conservativity of the production-destruction system irrespectively of the time step size chosen [1, 2, 3, 5, 6]. Due to these advantageous properties they are used for a wide variety of applications.

There has been a considerable interest in the development of MPRK schemes after being introduced in [7]. In [1, 2], MPRK schemes of second and third order were introduced. These were generalized in the context of SSP Runge-Kutta methods in [5, 6] and applied to solve reactive Euler equations. The idea of [7] was used to develop mPDeC schemes [8], which are MPRK schemes of arbitrary order based on deferred correction schemes. All these schemes are unconditionally positive and conservative and have proven their efficiency and robustness while integrating stiff PDS.

In [3], extended MPRK methods, named MPRKO methods, using Oliver's approach [9] to improve the accuracy of these schemes in the field of nonautonomous systems. The approach does not demand $\mathbf{Ae} = \mathbf{c}$ in the Butcher tableau $(\mathbf{A}, \mathbf{b}, \mathbf{c})$, where $\mathbf{e} = (1, \ldots, 1)^T$. Positivity and mass conservation fundamental properties were proven and even conditions concerning the Patankar weights were given to get second order accuracy of the MPRKO methods.

Third order modified Patankar–Runge–Kutta (MPRK) schemes were introduced in [2], which were developed to guarantee unconditional positivity and conservation, when integrating positive and conservative production-destruction systems. They introduced the first family of third-order MPRK schemes and its can be interpreted as four-stage methods named MPRK43 schemes. Recently, Kopecz and Meister proven that it is impossible to construct third-order MPRK schemes with only three stages, in the usual manner, which takes products of powers of previous stage values as Patankar-weight denominators [4].

In this paper, we continue our work in [3] and construct a third-order four-stage MPRK scheme for production–destruction equations. Following the lines of [2], the necessary and sufficient conditions for third order accuracy are derived and introduce one family of third order MPRKO methods.

Keywords:

Mathematics Subject Classifications (2010):

REFERENCES

[1] S. Kopecz, A. Meister. On order conditions for modified Patankar Runge–Kutta schemes. *Applied Numerical Mathematics*, 123: 159-179, 2018.
[2] S. Kopecz, A. Meister. Unconditional positive and conservative third order modified Patankar Runge Kutta discretizations of production destruction systems. BIT Num. Math., 58: 694-728, 2018.
[3] A. Ávila, G. González, S. Kopecz, A. Meister. Extension of modified Patankar–Runge–Kutta schemes to nonautonomous production–destruction systems based on Oliver's approach. *Jour. Compu. Appl. Math.*, 389: 20-33, 2020.
[4] S. Kopecz, A. Meister. On the existence of three-stage third-order modified Patankar–Runge–Kutta schemes. *Numer. Algorithms* 81: 1473-1484, 2019.

[5] J. Huang, CW. Shu. Positivity-preserving time discretizations for production–destruction equations with applications to non-equilibrium flows. *J. of Sci. Comp.* 78: 1811-1839, 2018.
[6] J. Huang, W. Zhao, CW. Shu. A Third-Order Unconditionally Positivity-Preserving Scheme for production-destruction Equations with Applications to Non-equilibrium Flows. J. of Sci. Comp. 79: 1015-1056, 2019.
[7] H. Burchard, E. Deleersnijder, and A. Meister. A high-order conservative Patankar-type discretisation for stiff systems of production-destruction equations. Appl. Numer. Math. 47: 1-30, 2003.
[8] P. Öffner, D. Torlo. Arbitrary high-order, conservative and positive preserving Patankar-type deferred correction schemes. Appl. Numer. Math. 153: 15-34, 2020.
[9] J. Oliver. A curiosity of low-order explicit Runge–Kutta methods. Math. Comput. 29: 1032–1036, 1975.

Universidad Central del Ecuador, Quito-Ecuador
Email address: gjgonzalez@uce.edu.ec

Universidad de la Frontera, Temuco, Chile
Email address: andres.avila@ufrontera.cl