# AN OPTIMIZATION BASED 3D-1D COUPLING FOR TISSUE PERFUSION AND CHEMICAL TRANSPORT IN GROWING CAPILLARY NETWORKS

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Abstract. The application of an optimization based 3D-1D coupling strategy is proposed for the simulation of tissue perfusion and chemical transport in evolving micro-vascular structures [1]. In particular the process of angiogenesis is taken into account, consisting in the formation of new capillaries from existing ones.

Angiogenesis simulations can become very complex and computationally expensive, especially if the blood flow inside the nascent vessels and the exchanges with the surrounding tissue are accounted for as the network grows. It is a common practice in the modeling of the interactions between a capillary network and the surrounding tissue to adopt a 3D-1D coupling strategy, in which, under proper assumptions on the regularity of the solution, the computational cost of the simulation is reduced by approximating the capillaries by their centerlines. The novelty of the proposed approach lies in the strategy adopted to solve the resulting 3D-1D set of equations [2]: two auxiliary variables are introduced, approximating the value of the unknowns on the capillary wall, and a properly designed cost functional is minimized constrained by the 3D-1D system of PDEs. This PDE-constrained optimization approach appears to be highly robust and flexible in handling geometrical complexities and hence it fits particularly well angiogenesis simulations. First of all, no conformity between the 3D and the 1D meshes is required, so that remeshing is never needed as the network grows. Furthermore, a proper choice of the interface variables yields a well conditioned discrete problem, regardless of the mutual sizes of the 3D and 1D partitions. Finally, interface variables are directly available, without the need of post processing. We propose the use of the optimization based 3D-1D coupling for the modeling of fluid pressure and oxygen concentration in the tissue and inside the growing capillaries. The model also accounts for the dispersion of a chemotactic growth factor (VEGF) which is responsible for the vessel growth and which is modeled by a 3D equation with a singular sink term.

**Keywords**: 3D-1D coupling, domain-decomposition, non conforming mesh, optimization methods for PDE problems, mathematical model of angiogenesis, fluid and chemical transport in evolving networks

**Mathematics Subject Classifications (2010)**: 65N30, 65N50, 68U20, 35Q92, 92B05, 92C17

## References

[1] S. Berrone, C. Giverso, D. Grappein, L. Preziosi, S. Scialò *An optimization based 3D-1D coupling strategy for tissue perfusion and chemical transport during tumor-induced angiogenesis*, (2023) https://arxiv.org/abs/2212.00692

[2] S. Berrone, D. Grappein, S. Scialò, *A PDE-constrained optimization method for 3D-1D coupled problems with discontinuous solutions*, Numerical Algorithms (2023), 10.1007/s11075-023-01579-w

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