PARAMETRIC SHAPE HOLOMORPHY OF BOUNDARY INTEGRAL OPERATORS: APPLICATIONS TO OPERATOR LEARNING AND MULTIFIDELITY BAYESIAN INVERSION

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Abstract. We consider a family of boundary integral operators supported on a collection of parametrically defined bounded Lipschitz boundaries. These operators depend on parametric variables, giving rise to a parameter-to-operator map. In the first part of this talk, we discuss the analytic or holomorphic dependence of said boundary integral operators upon the parametric variables, i.e., of the parameter-to-operator map. As a direct consequence we also establish holomorphic dependence of solutions to boundary integral equations, i.e., holomorphy of the parameter-to-solution map. The established parametric holomorphy results have been identified as a key property to derive best N-term approximation rates to overcome the so-called curse of dimensionality in the approximation of parametric maps with distributed, high-dimensional inputs.

In the second part of this talk, we illustrate the practical relevance of our findings by examining the sound-soft Helmholtz acoustic scattering problem and its frequency-robust boundary integral formulations. We are interested in exploring the relevance of this result in data-driven approaches to forward and inverse uncertainty quantification in acoustic scattering problems. More precisely, we consider:

(i) Data-driven learning of the parameter-to-solution map using the reduced basis method.
(ii) Multifidelity learning tailored for Bayesian problems, particularly in the small noise and large data limits.

We present a comprehensive mathematical analysis of both approaches, leveraging the parametric holomorphy result. Finally, we provide numerical experiments showcasing the advantages of these data-driven techniques over traditional methods in terms of efficiency, accuracy, and scalability.

Keywords: Parametric Holomorphy; Boundary Integral Operators; Operator Learning; Multifidelity Learning; Bayesian Inverse Problems.

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