STABLE ADAPTIVE LEAST-SQUARES SPACE-TIME BEM FOR THE WAVE EQUATION

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Abstract. We consider space-time boundary element methods for the weakly singular operator V corresponding to transient wave problems. In particular, we restrict ourselves to the one-dimensional case and work with prescribed Dirichlet data and zero initial conditions. We begin by revisiting two approaches: energetic BEM [**?**] and the more recent formulation proposed in [**?**], for which the weakly singular operator is continuous and satisfies inf-sup conditions in the related spaces. However, numerical evidence suggests that it is unstable when using low-order Galerkin-Bubnov discretisations. As an alternative, it was shown in [**?**] that one obtains ellipticity -and thus stability- by composing V with the modified Hilbert transform [**?**].

In this talk, we reformulate these variational formulations as minimisation problems in L^2. For discretisation, the minimisation problem is restated as a mixed saddle point formulation. Unique solvability can be established by combining conforming nested boundary element spaces for the mixed formulation such that the first-kind variational formulation is discrete inf-sup stable. We will analyse under which conditions the discrete inf-sup stability is satisfied, and, moreover, we will show that the mixed formulation provides a simple error estimator, which can be used for adaptivity. The theory is complemented by several numerical examples.

Keywords: Wave equation, Boundary Element Methods, Space-time, Least-squares

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