ON TIME STEPPING SCHEMES FOR THE DG DISCRETISATION OF FRIEDRICHS SYSTEMS. PART 1.

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Abstract. Symmetric Friedrichs systems constitute a large class of linear hyperbolic systems that englobe most of mathematical models for linear wave propagation phenomena: acoustics, electromagnetics, elastodynamics but also copled phenomena such as piezo-electricity or aeroacoustics that include convection phenomena due to acoustics-hydrodynamics.

A privileged method for the space discretisation of these models is the Discontinuous Galerkin Method whose field of application is much larger than the one of Finite Element Methods for instance. This method also offers a lot of flexibility allowing local mesh refinement and the treatment of hanging nodes. The use of off-centered numerical fluxes permits to introduce some artificial dissipation, which appears useful, for instance to control the spurious oscillations observed with centred fluxes (the conservative case) when modelling convection phenomena.

For time dependent simulations, the time discretisation of the semi-discrete problem issued from the DG method in both conservative and dissipative cases is obviously a key issue. For large scale simulations such as those generally encountered for wave propagation problems, explicit schemes must be privileged, which naturally raises the question of their stability. Curiously, this theoretical question does not seem to have been addressed extensively in the literature.

In these two presentations, we shall present various results in this direction for two classes of schemes : Runge Kutta type schemes and leap-frog type schemes. For the numerical analysis, two different techniques will be used : the Von Neumann approach (that provides optimal results but remains limited to particular cases) and the energy method.

Our work is related to three previous contributions in the literature. Concerning Von Neumann analysis, our results are quite close to those of the following paper on ODEs [1]. Concerning energy methods, our work has to be seen in the continuation of the following two previous works [3, 2].

Keywords: Discontinuous Galerkin methods, Runge-Kutta integrator, Leap frog scheme, stability, Friedrichs systems, energy method, Von Neumann analysis.

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