FULLY MIXED METHODS FOR THE COUPLED POROELASTICITY AND POISSON–NERNST–PLANCK EQUATIONS

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ABSTRACT. We introduce and analyze a Banach space-based approach yielding a fully-mixed finite element method for numerically solving the Biot poroelasticity and Poisson–Nernst–Planck equations. For the poroelasticity we consider a four-field formulation, whose primary variables are the solid displacement, the fluid pressure, the fluid flux, and the total pressure. In turn, besides the electrostatic potential and the concentration of ionized particles, we introduce as new unknowns its gradients and the total ionic fluxes. The resulting continuous formulation, posed in suitable Banach spaces, consists of a coupled system of two saddle point-type problems, for the poroelasticity and Poisson equations, and a twofold saddle-point problem for the ionized particles concentration equations. The well-posedness of it is then analyzed by applying the classical Banach fixed point theorem, along with a smallness assumption on the data, the Babuška–Brezzi theory in Banach spaces, and a slight variant of a recently obtained solvability result for perturbed saddle point formulations in Banach spaces as well. The associated Galerkin schemes are addressed similarly, and the Brouwer theorem yields the existence of discrete solutions. A priori error estimates are derived for both approaches, and rates of convergence for specific finite element subspaces satisfying the required discrete inf-sup conditions, are established in 2D. Finally, several numerical examples illustrating the performance of the two methods and confirming the theoretical findings, are reported.

Keywords: Biot problem; Poisson–Nernst–Planck problem; Mixed finite elements; Fixed point.

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