

CONSTRUCTION OF MULTIGRID SOLVERS FOR HYBRIDIZED DISCONTINUOUS GALERKIN DISCRETIZATIONS

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ABSTRACT. Since the approximation spaces of hybridized methods are not nested, the construction of multigrid injection operators is not trivial, see [CDGT13]. Starting from analyzing a particular injection operator in [LRK21], we derive general assumptions on HDG methods and injection operators in [LRK22a]. With these assumptions, we can prove contraction of multigrid iterations independent of the mesh size. We provide several examples of useful injection operators which fulfil these assumptions. By similar principles, we also show the convergence of two-level Schwarz methods in [LRK23]. In addition to standard HDG methods, the structure is applicable to embedded DG methods, where some aspects are even simpler [LRK22b].

Keywords: HDG, multigrid, GMG, injection operators

Mathematics Subject Classifications (2010): 65F10, 65N30, 65N50

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