

A ROBUST DISCONTINUOUS GALERKIN SCHEME ON ANISOTROPIC MESHES

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ABSTRACT. Discontinuous Galerkin (DG) methods are extensions of the usual Galerkin finite element methods. Although there are vast amount of studies on DG methods, most of them have assumed shape-regularity conditions on meshes for both theoretical error analysis and practical computations. In this paper, we present a new symmetric interior penalty DG scheme with a modified penalty term. We show that, without imposing the shape-regularity condition on the meshes, the new DG scheme inherits all of the good properties of standard DG methods, and is thus robust on anisotropic meshes. Numerical experiments confirm the theoretical error estimates obtained.

Keywords: discontinuous Galerkin method, symmetric interior penalty, error estimation, anisotropic meshes

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