PROXIMAL GALERKIN: A STRUCTURE-PRESERVING FINITE ELEMENT METHOD FOR POINTWISE BOUND CONSTRAINTS

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Abstract. The proximal Galerkin finite element method is a high-order, nonlinear numerical method that preserves the geometric and algebraic structure of bound constraints in infinite-dimensional function spaces. In this talk, we will introduce the proximal Galerkin method and apply it to solve free-boundary problems, enforce discrete maximum principles, and develop scalable, mesh-independent algorithms for optimal design. The proximal Galerkin framework is a natural consequence of the latent variable proximal point (LVPP) method: an unconditionally stable alternative to the interior point method. LVPP is an infinite-dimensional optimization algorithm that may be viewed as having an adaptive barrier function that is updated with a new informative prior at each (outer loop) optimization iteration. One of the main benefits of this algorithm is witnessed when analyzing the classical obstacle problem. Therein, we find that the original variational inequality can be replaced by a sequence of semilinear partial differential equations (PDEs) that are readily discretized and solved with, e.g., high-order finite elements. Throughout the talk, we will arrive at several unexpected contributions that may be of independent interest. These include (1) a semilinear PDE we refer to as the entropic Poisson equation; (2) an algebraic/geometric connection between high-order positivity-preserving discretizations and an infinite-dimensional Lie group; and (3) a gradient-based, bound-preserving algorithm for two-field density-based topology optimization. The complete latent variable proximal Galerkin methodology combines ideas from nonlinear programming, functional analysis, tropical algebra, and differential geometry and can potentially lead to new synergies among these areas as well as within variational and numerical analysis. This talk is based on [1].

Keywords: pointwise bound constraints, bound-preserving discretization, entropy regularization, proximal point

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References

[1] B. Keith, T.M. Surowiec. Proximal Galerkin: A structure-preserving finite element method for pointwise bound constraints arXiv preprint arXiv:2307.12444 2023.

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