A DIVERGENCE-FREE FINITE ELEMENT METHOD FOR FLOW-TRANSPORT COUPLING WITH OSMOTIC EFFECTS

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Abstract. The aim of this talk is to propose a model for the coupling of flow and transport equations with porous membrane-type conditions on part of the boundary. The governing equations consist of the incompressible Navier–Stokes equations coupled with an advection-diffusion equation, and we employ a Lagrange multiplier to enforce the coupling between penetration velocity and transport on the membrane while mixed boundary conditions are considered in the remainder of the boundary. Existence and uniqueness of the continuous problem using a fixed-point argument is proved. By means of a new H(div)-conforming finite element formulation, we address the model approximation, together with its a priori error analysis. We showcase a set of numerical examples validating the theory and illustrating the use of the new methods in the simulation of reverse osmosis processes.

Keywords: Navier–Stokes equations coupled with transport, Lagrange multipliers, Reverse osmosis, Divergence-conforming finite element methods.

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