THE ULTRA WEAK VARIATIONAL FORMULATION OF MAXWELL'S EQUATIONS

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ABSTRACT. The Ultra Weak Variational Formulation (UWVF) is a special Trefftz discontinuous Galerkin method due to Cessenat and Déspres [1]. Here we are concerned with solving the time-harmonic Maxwell's equations. The method uses superpositions of plane waves to represent solutions element-wise on a finite element mesh. Following a study of the method [**?**], we implemented a parallel UWVF version of the code, called *ParMax*. This has undergone development for industrial applications [3]. In this talk, we emphasize high-order solutions in the presence of scatterers with piecewise smooth boundaries. We explain the incorporation of curved surface triangles into the UWVF, necessitating quadrature for system matrix assembly. We also show how to implement the transmission conditions across an interface to handle resistive sheets. We note also that a wide variety of element shapes can be used, that the elements can be very large compared to the wavelength of the radiation, and that a matrix-free version is easy to implement (although computationally costly). Our contributions are illustrated through numerical examples demonstrating the efficiency enhancement achieved by curved elements in the UWVF. The method accurately handles resistive screens, as well as perfect electric conductor and penetrable scatterers. By employing large curved elements and the matrix-free approach, we successfully simulated X-band frequency scattering from an aircraft. These innovations demonstrate the practicality of the UWVF for industrial applications.

Keywords: Maxwell equations, Trefftz method, Numerical analysis, Frequency domain analysis

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