

RESOLVING SINGULARITIES IN PARABOLIC INITIAL-BOUNDARY VALUE PROBLEMS

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ABSTRACT. We consider a time-dependent reaction-diffusion equation with a singularity arising from incompatible initial and boundary conditions:

$$u_t - u_{xx} + b(x, t)u = f \quad \text{in } (0, \ell) \times (0, T],$$

subject to boundary conditions

$$u(0, t) = \phi_0(t), \quad u(\ell, t) = \phi_\ell(t), \quad t \in (0, T],$$

and the initial condition

$$u(x, 0) = 0, \quad x \in (0, \ell),$$

with $\phi_0(0) \neq 0$.

The discrepancy between initial and boundary conditions causes the formation of a singularity in the vicinity of the corner $(0, 0)$. This singularity s can be characterised as the solution of

$$s_t - s_{xx} + b(0, 0)s = 0 \quad \text{in } (0, \infty) \times (0, T],$$

subject to the boundary condition

$$s(0, t) = \phi_0(t), \quad t \in (0, T],$$

and the initial condition

$$s(x, 0) = 0, \quad x \in (0, \infty).$$

This in turn can be given analytically using the error function.

Now, the interesting question is: How can the remainder $y = u - s$ be resolved numerically?

We derive bounds on the derivative of y — under significantly less restrictive assumptions than previously assumed by other authors — and show how a numerical approximation can be obtained using an appropriately designed mesh.

Keywords: singularities, parabolic problems, adapted meshes

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