# THE ENO-ET SPATIAL RECONSTRUCTION SCHEME: ADVANCES IN NON-LINEAR HIGH-ORDER NUMERICAL SCHEMES

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Abstract. Non-linear spatial reconstruction is a successful procedure in circumventing Godunov's theorem and building high-order non-linear numerical methods for hyperbolic balance laws. The pioneering Essentially-Non-Oscillatory (ENO) method of Harten et al. [**?**, **?**, **?**] and the Weighted Essentially-Non-Oscillatory (WENO) method of Jiang and Shu [**?**] have been fundamental breakthroughs in providing the basis for constructing schemes that are both high-order accurate in smooth regions and essentially non-oscillatory at discontinuities.

Although the literature that shows prominent results is extensive, there is not yet a complete theory of convergence for these reconstruction schemes. Therefore procedures have to be analysed case by case. Problems of interest here, associated to ENO, include the difficulty to attain steady state solutions and the loss of accuracy in cases involving smooth solutions with steep gradients and abrupt stencil changes, [**?**].

In the present work we present a new method called ENO-ET, which is a variant of ENO. The scheme preserves the main advantages of ENO and successfully overcomes its well-known shortcomings. The method is assessed as a building block for high-order fully-discrete ADER numerical schemes [**?**, **?**]. The accuracy is extended up to the tenth order, not only to demonstrate its high order but also to identify features that become visible beyond the fifth order, the Gibbs phenomenon is one of them, which appears during the propagation of discontinuous profiles.

Results are presented in one and two dimensions and are compared with those from existing methods. Salient features include: i) attainment of theoretically expected convergence rates for smooth solutions arising from demanding initial conditions, ii) computation of essentially non-oscillatory profiles for discontinuous solutions, and (iii) successfully attainment of steady-steady state solutions through time marching.

**Keywords**: ENO reconstruction, WENO reconstruction, finite volume ADER schemes

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