ADAPTIVE PROJECTIONS IN DUAL NORMS: AN OVERVIEW OF THE STATE OF THE ART

IGNACIO MUGA, SERGIO ROJAS, KRISTOFFER G. VAN DER ZEE, AND PATRICK VEGA

Abstract. In [1], we proposed an adaptive projection method in negative Sobolev spaces $W^{-m,p}(\Omega)$ to regularize continuous linear functionals acting on regular Sobolev spaces such as $W^{m,q}(\Omega)$, where $m \geq 1$; $p^{-1} + q^{-1} = 1$; $1 < p < +\infty$; and Ω being a Lipschitz domain. In this context, regularization means to find an optimal piecewise polynomial function $\ell_H : \Omega \to \mathbb{R}$, such that the action of a functional ℓ over any test function v, can be approximated by $\langle \ell, v \rangle \approx \int_\Omega \ell_H v$. In this talk, we provide a state of the art of the method, including extensions/adaptations to nonconforming settings, new compatible pairs, projection of rougher linear functionals, duality-map localization features, and averaging operators, together with some numerical validations.

Keywords: Rough linear functionals, adaptive regularization, projection in dual norms, nonconforming discretizations.

Mathematics Subject Classifications (2010): 65N12, 65N15, 65N22, 65N30, 65N50.

References

[1] F. Millar, I. Muga, S. Rojas, and K. G. van der Zee. Projection in negative norms and the regularization of rough linear functionals. *Numerische Mathematik*, 150, 1087-1121, 2022.

[2] I. Muga, S. Rojas, K. G. van der Zee, and P. Vega. Source regularization through projection in dual norms for nonconforming finite element discretizations. *In preparation*, 2023.

Instituto de Matemáticas, Pontificia Universidad Católica de Valparaíso, Chile
Email address: ignacio.muga@pucv.cl

Instituto de Matemáticas, Pontificia Universidad Católica de Valparaíso, Chile
Email address: sergio.rojas.h@pucv.cl

School of Mathematical Sciences, University of Nottingham, UK
Email address: KG.vanderZee@nottingham.ac.uk

Departamento de Matemática y Ciencia de la Computación, Universidad de Santiago de Chile, Chile.
Email address: patrick.vega@usach.cl