SOURCE REGULARIZATION THROUGH PROJECTION IN DUAL NORMS FOR NONCONFORMING FINITE ELEMENT DISCRETIZATIONS

IGNACIO MUGA, SERGIO ROJAS, KRISTOFFER G. VAN DER ZEE, AND PATRICK VEGA

Abstract. We propose an adaptive projection method for the regularization of continuous linear functionals acting on Sobolev spaces. We focus on projections onto finite element spaces, computing the best approximation measured in a dual norm. Irregular functionals act in more regular spaces, so the number of degrees of freedom associated with conforming finite element spaces increases, motivating the use of nonconforming discretizations as an extension of their conforming counterpart [1]. Working with nonconforming finite elements is non-trivial if the source term is not in the dual of the finite element space, making our approach particularly useful in this context [2], since the regularization belongs to such a dual space. This approach also computes a residual representative as an additional unknown in an equivalent mixed variational formulation, which, together with the projections, are computed through an adaptive mesh refinement procedure driven by the residual representative. For computability reasons, we use a discrete dual norm that supremizes over a nonconforming finite element space. We propose nonconforming compatible pairs and, therefore, obtain quasi-optimal convergent methods. Finally, we show how our projection method can efficiently compute nonconforming approximations to partial differential equations with (ultra) rough data.

Keywords: projection in dual norms, adaptive regularization, a posteriori error analysis, residual minimization, nonconforming discretizations.

Mathematics Subject Classifications (2010): 65N12, 65N15, 65N22, 65N30, 65N50.

Funding: IM was supported by the Chilean National Agency for Research and Development (ANID) through FONDECYT project 1230091. SR was supported by the Chilean grant ANID FONDECYT 3210009. K. G. vdZ was partially funded by the Engineering and Physical Sciences Research Council (EPSRC), UK, under Grant EP/W010011/1. PV was funded by the Chilean grant ANID FONDECYT 3220858.

References

[1] F. Millar, I. Muga, S. Rojas, and K. G. van der Zee. Projection in negative norms and the regularization of rough linear functionals. *Numerische Mathematik*, 150, 1087-1121, 2022.

[2] I. Muga, S. Rojas, K. G. van der Zee, and P. Vega. Source regularization through projection in dual norms for nonconforming finite element discretizations. *In preparation*, 2023.

Instituto de Matemáticas, Pontificia Universidad Católica de Valparaíso, Chile
Email address: ignacio.muga@pucv.cl

Instituto de Matemáticas, Pontificia Universidad Católica de Valparaíso, Chile
Email address: sergio.rojas.h@pucv.cl

School of Mathematical Sciences, University of Nottingham, UK
Email address: KG.vanderZee@nottingham.ac.uk

Departamento de Matemática y Ciencia de la Computación, Universidad de Santiago de Chile, Chile.
Email address: patrick.vega@usach.cl