SYMPLECTIC HAMILTONIAN HYBRIDIZABLE DISCONTINUOUS GALERKIN METHODS FOR LINEARIZED SHALLOWS-WATER EQUATIONS

CRISTHIAN NÚÑEZ AND MANUEL A. SÁNCHEZ

Abstract. We focus on devising novel numerical methods for solving Linearized Shallow-Water Equations (LSWE) while preserving physical quantities of interest such as energy, vorticity, among others. The Shallow-Water Equations describe the dynamics of a fluid with constant density at low depths, which are entirely determined by the conservative laws for momentum and mass. We employ a suitable rewriting of LSWE in a Hamiltonian form and utilize Hibridizable Discontinuous Galerkin Methods (HDG) for spatial discretization. This approach leverages the Hamiltonian structure of SWE combined with the symplectic time-stepping methods such us SDIRK, which conserve the discrete energy of the system over time. We discuss the main properties of our methodology and present numerical experiments to verify its performance, comparing it with other dissipative approaches found in the bibliography.

Keywords: Shallow-Water, HDG, Hamiltonian, symplectic time-stepping.

Mathematics Subject Classifications (2010): 65P10

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Pontificia Universidad Católica de Chile, Facultad de Matemáticas, Santiago, Chile
Email address: aanunez6@uc.cl

Pontificia Universidad Católica de Chile, Instituto de Ingeniería Matemática y Computacional IMC, Santiago, Chile
Email address: manuel.sanchez@uc.cl