NUMERICAL SIMULATION OF WAVE PROPAGATION IN A WAVEGUIDE USING TREFFTZ ELEMENTS

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Abstract. We investigate numerically the propagation of time-harmonic waves along an unbounded waveguide for medium and large piecewise constant wavenumbers. We do so based on a Trefftz Discontinuous Galerkin (TDG) formulation which we discretize with the superposition of travelling plane waves.

We first rewrite the problem on a bounded computational domain by using the Neumann-to-Dirichlet map on the artificial walls. This truncated problem is formulated variationally in a DG way, that is, the interelement continuity is imposed weakly within the variational formulation by introducing suitable numerical fluxes. We choose standard numerical fluxes for internal faces and some more exotic numerical fluxes for faces on the truncation boundary. We then get a consistent and coercive formulation which achieves quasi-optimal convergence when discretized with Trefftz elements. We also provide a priori error bounds for the discretization based on plane waves.

The behavior of the numerical solutions and their order of convergence is verified and illustrated in two dimensions with numerical experiments, including for the special case of the Ultra Weak Variational Formulation (UWVF). In particular, these experiments allow us to investigate the instability and ill-conditioning inherent in plane wave-based Trefftz methods, and if this issue can be overcome in practice with suitable regularization techniques.

Keywords: time-harmonic wave, waveguide, discontinuous Galerkin formulation, ultra weak variational formulation, Trefftz elements

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