HDG METHODS WITH TRANSMISSION VARIABLES FOR HELMHOLTZ PROBLEMS

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Abstract. Due to their ability to deliver high-order convergence on unstructured meshes, discontinuous Galerkin finite element methods are well suited for time-harmonic wave propagation problems, where the Helmholtz equation is solved on a variety of complicated physical (*e.g.* with regard to heterogeneous media) and geometrical domains. However, the associated linear system is often very large and poorly conditioned. This renders the use of direct solvers too costly and the convergence of iterative schemes slow.

To overcome these difficulties, a hybridizable discontinuous Galerkin method (CHDG) [**?**] is proposed. It is based on a standard discontinuous Galerkin scheme with upwind numerical fluxes and high-order polynomial bases. Auxiliary unknowns corresponding to characteristic variables are defined at the interface across the elements, so that the physical fields inside each element are eliminated in order to obtain a reduced system. Eventually, the reduced system can be written in terms of a fixed-point problem that can be solved with stationary iterative schemes, such as CGNR, GMRES and fixed-point.

Numerical results for 2D benchmarks are presented to study the performance of the approach. Compared to the standard HDG approach, the properties of the reduced system are improved with CHDG, which is therefore better suited for iterative solution procedures: the condition number of the reduced system is smaller and both CGNR and GMRES require smaller numbers of iterations.

We are currently investigating extended methods to exploit high-order transmission variables to further reduce the number of iterations (inspired by [**?**]), as well as to deal with heterogeneous media. Preliminary theoretical and numerical results will be presented.

Keywords: Helmholtz equation, Discontinuous finite elements, Hybridization, Iterative solvers.

Mathematics Subject Classifications (2010): 35J05, 65N12, 65N30

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