SYMPLECTIC HAMILTONIAN FINITE ELEMENT METHODS FOR SEMILINEAR WAVE PROPAGATION

MANUEL A. SÁNCHEZ AND JOAQUÍN VALENZUELA

ABSTRACT. This paper presents Hamiltonian finite element methods for approximating a class of semilinear wave propagation problems, including the nonlinear Klein-Gordon and sine-Gordon equations. The aim is to obtain accurate high-order approximations while conserving physical quantities of interest such as energy. To achieve conservation properties at a discrete level, we propose semidiscrete schemes based on two Hamiltonian structures of the equation. These include Mixed finite element methods, discontinuous Galerkin methods, and hybridizable discontinuous Galerkin methods (HDG). In particular, we propose a new class of DG methods using time operators to define the numerical traces resulting in an energy-conserving scheme. Symplectic explicit-partitioned, and diagonally-implicit Runge Kutta schemes are used for time discretization. We present numerical examples illustrating the approximations' accuracy and energy conservation of the approximations and the simulation of soliton cloning.

Keywords: hybridizible discontinuous Galerkin, Hamiltonian systems, semilinear wave

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(1) INSTITUTE FOR MATHEMATICAL AND COMPUTATIONAL ENGINEERING, PONTIFICIA UNIVERSIDAD CATÓLICA DE CHILE, SANTIAGO, CHILE
Email address: manuel.sanchez@uc.cl

(J. Valenzuela) SCHOOL OF ENGINEERING, PONTIFICIA UNIVERSIDAD CATÓLICA DE CHILE, SANTIAGO, CHILE
Email address: javalenzuela3@uc.cl

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