# WELL-POSEDNESS AND NUMERICAL ANALYSIS OF AN ELAPSED TIME MODEL WITH STRONGLY COUPLED NEURAL NETWORKS

MAURICIO SEPÚLVEDA, NICOLAS TORRES, AND LUIS MIGUEL VILLADA

Abstract. The elapsed time equation is an age-structured model that describes dynamics of interconnected spiking neurons through the elapsed time since the last discharge, leading to many interesting questions on the evolution of the system from a mathematical and biological point of view. In this work, we first deal with the case when transmission after a spike is instantaneous and the case when there exists a distributed delay that depends on previous history of the system, which is a more realistic assumption. Then we study the well-posedness and the numerical analysis of the elapsed time models. For existence and uniqueness we improve the previous works by relaxing some hypothesis on the non-linearity, including the strongly excitatory case, while for the numerical analysis we prove that the approximation given by the explicit upwind scheme converges to the solution of the non-linear problem. We also show some numerical simulations to compare the behavior of the system in the case of instantaneous transmission with the case of distributed delay under different parameters, leading to solutions with different asymptotic profiles.

**Keywords**: Structured equations; Mathematical neuroscience; Delay differential equations, Well-posedness, Numerical analysis; Periodic solutions.

**Mathematics Subject Classifications (2010)**: 35A35, 35F20, 35R09, 65M06.

## References

[1] Pakdaman K, Perthame B, Salort D. Dynamics of a structured neuron population. *Nonlinearity.* 2009;23(1):55

[2] Torres N, Cáceres MJ, Perthame B, Salort D. An elapsed time model for strongly coupled inhibitory and excitatory neural networks. *Physica D: Nonlinear Phenomena.* 2021:132977

CI$^2$MA and DIM, Universidad de Concepción, Concepción-Chile
*Email address*: maursepu@udec.cl

Departamento de Matemática Aplicada, Universidad de Granada-España
*Email address*: torres@ugr.es

GIMNAP-Departamento de Matemática, Universidad del Bío-Bío and CI$^2$MA-Universidad de Concepción, Concepción-Chile
*Email address*: lvillada@ubiobio.cl

This work has been supported by ANID project ECOS200018. MS and LMV was partially supported by ANID-Chile through Centro de Modelamiento Matemático (FB210005) and INRIA Associated team ANACONDA. MS was supported by Fondecyt-ANID project 1220869, and Jean d'Alembert fellowship program, Université de Paris-Saclay. NT was supported by the grant Juan de la Cierva FJC2021-046894-I funded by MCIN/AEI and the European Union NextGenerationEU/PRTR..