NUMERICAL ANALYSIS OF THE LANDAU–LIFSHITZ–BARYAKHTAR EQUATION IN MICROMAGNETICS

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Abstract. The Landau–Lifshitz equation is commonly used in micromagnetics to model the effects of a magnetic field on ferromagnetic materials [4]. An important feature of this equation is the conservation of magnitude of the magnetisation vector. It is known that this equation is essentially valid only at very low temperature, since it completely ignores contributions from high-frequency spin waves responsible for longitudinal magnetisation fluctuation [1]. To rectify this problem, the Landau–Lifshitz–Baryakhtar (LLBar) [2] and the Landau–Lifshitz–Bloch (LLBloch) [3] equations were proposed in the physics literature. These models take into account longitudinal relaxation and are valid at high temperatures, which are important for applications, for instance in heat-assisted magnetic recording and magnonic devices.

Existence and uniqueness of global weak and strong solutions to the LLBar equation were proven in [6], while existence of global weak solutions to the LLBloch equation were shown in [5]. We proposed some fully discrete numerical schemes to solve the LLBar equation, including a method based on C^1-conforming finite element [7] and another based on mixed formulation of the equation [8], in each case obtaining optimal order of convergence to the solution. As a by-product of our analysis, we showed the convergence of solution of the LLBar equation to the LLBloch equation (for high temperature but still below the Curie temperature) and the uniqueness of weak solution to the LLBloch equation.

Keywords: Landau–Lifshitz, Landau–Lifshitz–Baryakhtar, micromagnetics, finite element.

Mathematics Subject Classifications (2010): 65M12, 65M60, 35K35, 35Q60.

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