MULTISYMPLECTIC HYBRID FINITE ELEMENT METHODS FOR HAMILTONIAN PDES

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ABSTRACT. Solutions to Hamiltonian PDEs satisfy a local conservation law, called the *multisymplectic conservation law*, which generalizes the symplectic conservation law for Hamiltonian ODEs. We discuss a class of hybrid finite element methods for Hamiltonian PDEs—including conforming, nonconforming, mixed, and HDG methods—whose numerical solutions satisfy this conservation law locally (i.e., element-by-element) in terms of the numerical trace/flux. This includes methods for canonical Hamiltonian systems in mixed div-grad form [1, 2], as well as a more recent generalization to hybrid methods in finite element exterior calculus [3].

Keywords: hybrid finite element methods, hybridizable discontinuous Galerkin (HDG), Hamiltonian, multisymplectic, structure-preserving, finite element exterior calculus (FEEC)

Mathematics Subject Classifications (2010): 65N30, 37K05

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