AN INCOMPLETE BALANCING DOMAIN DECOMPOSITION METHOD BASED ON POLYNOMIAL FINITE ELEMENT SPACES

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ABSTRACT. A Balancing Domain Decomposition (BDD) method is originally proposed by Mandel [?], and is regarded as the preconditioner of linear iterative solvers for artificial boundary problems appearing in Domain Decomposition Methods (DDM) based on finite element methods; see, for example, Glowinski, et al. [?]. There exist many literatures on the application of BDD into large-scale computational models; see, for example in case of elastic problems, Ogino, et al. [?].

On the other hand, we have already computed ultra large-scale computational models derived from finite element methods for magnetic field problems by using iterative DDM; see, for example, Sugimoto, et al. [?]. However, in [?], we have used a simplified diagonalization as the preconditioner. Therefore, we require more effective preconditioner to reduce computational costs much more, and try to apply a BDD method into the preconditioner of an iterative DDM for magnetic field problems, where the magnetic vector potential is regarded as an unknown function.

When using a BDD method, we need to set a coarse space, which includes the kernel of the coefficient matrix of resultant linear system derived from a corresponding artificial boundary problem. In case of magnetic field problems, the number of the Degrees Of Freedom (DOF) of the coarse space is equal to the number of nodal points of triangulation. This fact leads BDD methods for magnetic field problems can keep the condition number of the coefficient matrix of resultant linear system, but we cannot expect to reduce their computational costs.

Now, to reduce the number of DOF of coarse spaces in BDD methods and their computational costs, Polytopal Element Methods (PEM; see Di Pietro, et al. [?]) is introduced, and is used for approximations of the coarse space with Domain-by-Domain methods. We call the method *incomplete BDD.* Owing to the approximation of coarse spaces, we can expect to reduce computational costs to solve coarse-space problems as well as to keep the condition number of the coefficient matrix of resultant linear system.

Keywords: balancing domain decomposition, discrete de Rham sequence, polytopal element, perturbed magnetostatic problem, polyhedral mesh

Mathematics Subject Classifications (2010): 65N55, 78M10, 78M25

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