BOUNDARY ELEMENT METHODS FOR FOCUSED ULTRASOUND TREATMENT IN BIOMEDICAL ENGINEERING

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Abstract. Focused Ultrasound (FUS) is a non-invasive treatment technique where acoustic energy is transmitted into the human body. The high acoustic pressure in the focal region causes ablation of tissue. Computational simulations can aid in the design of ultrasound instruments and the patient-specific treatment planning, guiding sufficient energy towards to lesion while avoiding overheating of healthy tissue, organs and bone. The adoption of computational approaches hinges on their accuracy in modelling acoustic transmission at high frequencies through realistic geometries that model the targeted regions. The boundary element method (BEM) is a powerful algorithm to solve the Helmholtz equation for harmonic acoustic waves. However, standard approaches fail to efficiently model realistic FUS applications. This talk presents several algorithmic improvements, such as OSRC preconditioning [4], nested meshes for volume integrals [2], high-contrast formulations [5], FEM-BEM coupling [6], and nonconforming meshes [7]. Hundreds of boundary integral formulations were benchmarked to provide efficiency guidelines [3]. We used our fast and accurate BEM implementation to simulate FUS in the human body [1], which can be translated to important biomedical applications such as the non-invasive treatment of liver cancer and neuromodulation of the brain. We validated the methodology within the benchmarking exercise of the International Transcranial Ultrasonic Stimulation Safety and Standards (ITRUSST) consortium [8]. Finally, we implemented all functionality in our open-source Python library, OptimUS.

Keywords: Boundary Element Method, Integral Equations, Acoustics

Mathematics Subject Classifications (2010): 65M38, 35J08

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