ON THE CONVERGENCE OF A SECOND-ORDER SCHEME FOR NON-LOCAL CONSERVATION LAWS

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Abstract. In this talk, we discuss the convergence analysis of a second-order numerical scheme for traffic flow models that incorporate non-local conservation laws. In the field of computational fluid dynamics, first-order methods are generally considered robust and reliable, and they aid in establishing well-posedness of problems. However, second- or high-order methods offer the advantage of considerably more accurate solutions with the same computing cost, particularly for two- or three-dimensional problems. As a result, there has been a surge of research activities aimed at improving these high-order methods. Those studies suggest that high-order schemes offer better solutions than the low-order schemes. In our work, we combine a MUSCL-type spatial reconstruction with strong stability preserving Runge-Kutta time-stepping to devise a fully discrete second-order scheme. The resulting scheme is shown to converge to a weak solution by establishing the maximum principle, bounded variation estimates and L^1 Lipschitz continuity in time. Further, using a space-step dependent slope limiter, we prove its convergence to the entropy solution. We also propose a MUSCL-Hancock type second-order scheme which requires only one intermediate stage unlike the Runge-Kutta schemes and is easier to implement. The performance of the proposed second-order schemes in comparison to a first-order scheme is demonstrated through several numerical experiments.

Keywords:Non-local conservation laws, MUSCL method, Second-order scheme, MUSCL-Hancock scheme, Convergence analysis, Entropy solution.

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