

# CURVATURE AND THE HHJ METHOD

SHAWN W. WALKER

ABSTRACT. This talk presents some recent advances in extending the classic Hellan–Herrmann–Johnson (HHJ) finite element to surfaces for approximation of bending problems and computing curvature. We give a brief review of the surface version of the HHJ method which leads to a convergent method to solve the surface Kirchhoff plate problem on surfaces embedded in  $\mathbb{R}^3$ , along with numerical examples. We also describe a post-processing technique for approximating the surface Hessian of a scalar function from discrete data. We show how this scheme is easily extended to give convergent approximations of the *full shape operator* of the underlying surface, even for piecewise linear triangulations. Several numerical examples are given on non-trivial surfaces to illustrate the method. Lastly, we describe on-going work on how the surface HHJ scheme can be modified for computing Willmore flow.

**Keywords:** surface finite elements, surface Hessian, Kirchhoff plate, bending, shape operator, non-conforming method.

**Mathematics Subject Classifications (2010):** 65D18, 65N30, 35J40

## REFERENCES

- [1] Douglas N. Arnold and Shawn W. Walker. The Hellan–Herrmann–Johnson Method with Curved Elements. *SIAM Journal on Numerical Analysis*, 58(5):2829–2855, 2020.
- [2] Shawn W. Walker. The Kirchhoff Plate Equation on Surfaces: The Surface Hellan–Herrmann–Johnson Method. *IMA Journal of Numerical Analysis*, 42(4):3094–3134, 2021.
- [3] Shawn W. Walker. Approximating the Shape Operator with the Surface HHJ Element. *in review*, 2023.

LOUISIANA STATE UNIVERSITY  
Email address: [walker@math.lsu.edu](mailto:walker@math.lsu.edu)