A SIMPLE AND EFFICIENT CONVEX OPTIMIZATION BASED BOUND-PRESERVING HIGH ORDER ACCURATE LIMITER FOR CAHN–HILLIARD–NAVIER–STOKES SYSTEM

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Abstract. For time-dependent PDEs, the numerical schemes can be rendered bound-preserving without losing conservation and accuracy, by a post processing procedure of solving a constrained minimization in each time step. Such a constrained optimization can be formulated as a nonsmooth convex minimization, which can be efficiently solved by first order optimization methods, if using the optimal algorithm parameters. By analyzing the asymptotic linear convergence rate of the generalized Douglas–Rachford splitting method, optimal algorithm parameters can be approximately expressed as a simple function of the number of out-of-bounds cells. We demonstrate the efficiency of this simple choice of algorithm parameters by applying such a limiter to cell averages of a discontinuous Galerkin scheme solving phase field equations for 3D demanding problems. Numerical tests on a sophisticated 3D Cahn–Hilliard–Navier–Stokes system indicate that the limiter is high order accurate, very efficient, and well-suited for large-scale simulations. For each time step, it takes at most 20 iterations for the Douglas–Rachford splitting to enforce bounds and conservation up to the round-off error, for which the computational cost is at most $80N$ with N being the total number of cells.

Keywords: Douglas–Rachford splitting, nearly optimal parameters, bound-preserving limiter, discontinuous Galerkin method, Cahn–Hilliard–Navier–Stokes, high order accuracy

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